

Relic Neutrino Asymmetries¹

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Abstract. I review the topic of relic neutrino asymmetry generation through active-sterile neutrino and antineutrino oscillations in the early universe. Applications to (i) the suppression of sterile neutrino production, and (ii) the primordial Helium abundance are briefly presented.

Reasonably large relic neutrino asymmetries will be generated by active-sterile neutrino and antineutrino oscillations in the early universe provided that certain mild conditions are met [1]. “Reasonably large” in this context means that the neutrino asymmetry, defined by

$$L_{\nu_\alpha} \equiv \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma}, \quad (1)$$

where $\alpha = e, \mu, \tau$ and the n ’s are number densities, can have a final value as large as about 3/8 [2,3]. The mild conditions are that the vacuum mixing angle θ_0 should be small and that the Δm^2 should be negative. The flavour and mass eigenstates are related by

$$\begin{aligned} |\nu_\alpha\rangle &= \cos\theta_0|\nu_a\rangle + \sin\theta_0|\nu_b\rangle \\ |\nu_s\rangle &= -\sin\theta_0|\nu_a\rangle + \cos\theta_0|\nu_b\rangle, \end{aligned} \quad (2)$$

so $\Delta m^2 < 0$ means that the predominantly sterile mass eigenstate is lighter than the predominantly active mass eigenstate. We focus on that epoch of the early universe subsequent to the disappearance of a significant muon/antimuon component to the plasma, and ending with the period of Big Bang Nucleosynthesis (BBN).

One reason the generation of neutrino or lepton asymmetries is interesting is evident from the effective matter potential,

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$$V_{eff} = \frac{\Delta m^2}{2p}(-a + b) \quad (3)$$

where p is the neutrino momentum, and

$$a \equiv -\frac{4\sqrt{2}\zeta(3)}{\pi^2} \frac{G_F T^3 p}{\Delta m^2} L^{(\alpha)}, \quad (4)$$

$$b \equiv -\frac{4\sqrt{2}\zeta(3)}{\pi^2} A_\alpha \frac{G_F T^4 p^2}{m_W^2 \Delta m^2}. \quad (5)$$

The function a is just the generalisation of the usual Wolfenstein effective potential [4] pertinent to the considered epoch of the early universe. G_F is the Fermi constant, T is the temperature and the “effective asymmetry” $L^{(\alpha)}$ is given by

$$L^{(\alpha)} \equiv L_{\nu_\alpha} + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} + \eta, \quad (6)$$

where $\eta \sim 10^{-10} - 10^{-9}$ is related to the baryon and electron asymmetries required to produce the universe we observe. When lepton asymmetries are large, the effective potential is also large, leading to small matter-affected mixing angles and thus also to the suppression of sterile neutrino production. The asymmetry generation effect must be taken into account when exploring the implications for BBN of neutrino scenarios involving light sterile degrees of freedom. In particular, $\nu_\mu \leftrightarrow \nu_s$ oscillation parameters motivated by the atmospheric neutrino deficit are far from being necessarily ruled out by BBN [1,2,5,6]. The function b is the leading finite- T correction to the Wolfenstein term [7]. The constant m_W is the W -boson mass, while A_α is a numerical factor given by $A_e \simeq 17$ and $A_{\mu,\tau} \simeq 4.9$. At high temperatures, b is large enough to suppress sterile neutrino production by itself. However, it decreases with temperature as T^6 , so a large asymmetry is eventually required to keep sterile neutrino production suppressed at lower temperatures.

Another reason lepton asymmetries are interesting is conveyed by the reactions,

$$\nu_e n \leftrightarrow e^- p, \quad \bar{\nu}_e p \leftrightarrow e^+ n, \quad (7)$$

which play an important role in setting the n/p ratio at weak freeze-out just prior to BBN. A sufficiently large electron neutrino asymmetry (a few percent) will alter this ratio and hence change the predicted Helium abundance [3,8,9]. In particular, successful BBN can be achieved with a higher than usual baryon density, provided that L_{ν_e} is positive and of the correct magnitude. The threatened overproduction of Helium is compensated by the preferential conversion of neutrons into protons due to the positive L_{ν_e} .

Light sterile neutrinos are themselves of considerable interest independent of their possible role in asymmetry generation. For instance, two-fold maximal mixing is easily achieved through a pseudo-Dirac structure [10] or through ordinary-mirror neutrino mixing [11]. (Mirror neutrinos are strictly speaking not sterile because they feel mirror weak interactions. However, they are effectively sterile from the

point of view of ordinary weak interactions.) Two-fold maximal mixing is of great interest in light of the solar and atmospheric neutrino deficits. In addition, it is well known that least one light sterile flavour is required if oscillations are to simultaneously explain the solar, atmospheric and LSND neutrino data.

As a final comment about motivation, let me also add that collision and matter affected neutrino oscillation dynamics is a fascinating subject in its own right. As we will see, neutrino asymmetry generation is driven by a subtle interplay between quantal coherence and decoherence.

Consider for simplicity a two-flavour active-sterile system $\nu_\alpha + \nu_s$ together with its antiparticle counterpart. The appropriate dynamical variables are the 1-body reduced density matrices for the neutrinos and antineutrinos,

$$\rho = \frac{1}{2}(P_0 + \vec{P} \cdot \sigma), \quad \bar{\rho} = \frac{1}{2}(\bar{P}_0 + \vec{\bar{P}} \cdot \sigma), \quad (8)$$

respectively. The decomposition with respect to the 2×2 identity matrix and the Pauli matrices proves to be convenient. All of these quantities are functions of neutrino momentum p and time t (or equivalently temperature T).

The diagonal entries ρ are appropriately normalised distribution functions for ν_α and ν_s :

$$\begin{aligned} \rho_{\alpha\alpha} &= \frac{1}{2}(P_0 + P_z) = \frac{N_\alpha}{N^{eq}(0)}, \\ \rho_{ss} &= \frac{1}{2}(P_0 - P_z) = \frac{N_s}{N^{eq}(0)}, \end{aligned} \quad (9)$$

where $n_f = \int_0^\infty N_f dp$ and $N^{eq}(\xi)$ is the Fermi-Dirac (FD) distribution with dimensionless chemical potential $\xi \equiv \mu_{\nu_\alpha}/T$,

$$N^{eq}(\xi) = \frac{1}{2\pi^2} \frac{p^2}{\exp(\frac{p}{T} - \xi) + 1}. \quad (10)$$

We have chosen to normalise the neutrino distribution to the FD distribution with zero chemical potential.

The off-diagonal entries,

$$\rho_{\alpha s} = \rho_{s\alpha}^* = \frac{1}{2}(P_x - iP_y), \quad (11)$$

are *coherences*. They quantify the amount of quantal or phase coherence enjoyed by the neutrinos. These quantities are needed because non-forward neutrino scattering off the background plasma decreases quantal coherence. The entries of $\bar{\rho}$ have a similar interpretation.

The evolution of ρ is given by the Quantum Kinetic (or Boltzmann) Equations (QKEs) [12],

$$\begin{aligned}\frac{\partial \vec{P}}{\partial t} &= \vec{V} \times \vec{P} - D\vec{P}_T + \frac{\partial P_0}{\partial t} \vec{z}, \\ \frac{\partial P_0}{\partial t} &\simeq \Gamma \left[\frac{N^{eq}(\xi)}{N^{eq}(0)} - \frac{1}{2}(P_0 + P_z) \right],\end{aligned}\tag{12}$$

where $D = \Gamma/2$ and $\vec{P}_T \equiv P_x \vec{x} + P_y \vec{y}$ with $\vec{x}, \vec{y}, \vec{z}$ being unit vectors in the stated directions. The second equation above has an approximate equality sign because the righthand side assumes that all background fermions have thermal FD distributions, and that ν_α is distributed in an approximately thermal manner.

The $\vec{V} \times \vec{P}$ term is just a re-expression of coherent matter-affected oscillatory neutrino evolution. It is equivalent to the usual Schrödinger Equation governing matter-affected neutrino oscillations. Note, however, that in the early universe this evolution is non-linear because of neutrino scattering off the background neutrinos of the same flavour. The vector \vec{V} is given by

$$\vec{V} = \beta \vec{x} + \lambda \vec{z},\tag{13}$$

where

$$\beta = \frac{\Delta m^2}{2p} \sin 2\theta_0, \quad \lambda = -\frac{\Delta m^2}{2p} \cos 2\theta_0 + V_{eff}.\tag{14}$$

The non-linear effects enter through V_{eff} .

The $-D\vec{P}_T$ term drives *collisional quantal decoherence*. The decoherence rate is equal to half of the collision rate Γ for a neutrino of momentum p , where

$$\Gamma \simeq y_\alpha G_F^2 T^5 \frac{p}{\langle p \rangle}.\tag{15}$$

The quantity $\langle p \rangle \simeq 3.15T$ is the average momentum for a FD distribution with zero chemical potential, while $y_e \simeq 4$ and $y_{\mu,\tau} \simeq 2.9$. This expression for Γ is correct provided that thermal equilibrium holds and asymmetries are not very large. The $-D\vec{P}_T$ term tries to exponentially damp the coherences $P_{x,y}$ to zero. Since the collision rate goes as T^5 , quantal coherence is expunged at high temperatures. In typical applications, the $\vec{V} \times \vec{P}$ term begins to dominate over the $-D\vec{P}_T$ term as T approaches a few MeV.

The $\partial P_0/\partial t$ equation describes the repopulation of the ν_α distribution from the background plasma heat bath. It is proportional to the total weak collision rate Γ , and the term in square brackets on the righthand side acts to drive the actual ν_α distribution function $(P_0 + P_z)N^{eq}(0)/2$ to FD form $N^{eq}(\xi)$.

The neutrino asymmetry L_{ν_α} enters these equations through the function a in V_{eff} , and through the chemical potential ξ in $\partial P_0/\partial t$. For small L_{ν_α} ,

$$L_{\nu_\alpha} \simeq \frac{T^3}{6n_\gamma} \xi.\tag{16}$$

In thermal equilibrium the existence of a neutrino asymmetry is equivalent to the existence of a nonzero ξ (where the chemical potential for antineutrinos is equal and opposite that of the neutrinos above the chemical decoupling temperature).

The antineutrino QKEs are of the same form as the above equations with the substitutions $L_{\nu_\alpha} \rightarrow -L_{\nu_\alpha}$ and $L^{(\alpha)} \rightarrow -L^{(\alpha)}$.

We are primarily interested in the evolution of the asymmetry L_{ν_α} . This is indirectly given by the neutrino and antineutrino QKEs. It is easy enough, however, to derive a redundant but useful direct evolution equation for the asymmetry. Employing the QKEs and $\alpha + s$ lepton number conservation one obtains

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{1}{2n_\gamma} \int_0^\infty \beta(P_y - \bar{P}_y) N^{eq}(0) dp. \quad (17)$$

This equation is numerically useful because, during most of its evolution, L_{ν_α} is the difference of two large numbers (the neutrino and antineutrino number densities per photon). The use of Eq.(17) circumvents this numerical difficulty. This equation is also a very useful starting point for developing approximate evolution equations [2,13].

We now discuss the behaviour of the QKEs. We will suppose that the initial asymmetries are very small or zero, and that a negligible fraction of the primordial plasma is in the form of sterile states. Also, we consider $|\Delta m^2|$ values higher than about 10^{-4} eV^2 , because for lower values asymmetry growth begins when decoherence is negligible, and it tends to be oscillatory [14].

For sufficiently high temperatures T , neutrino and antineutrino oscillations are severely damped or frozen. There are two reasons for this. First, the decoherence function $D \sim T^5$ is large and drives $P_{x,y}$ and $\bar{P}_{x,y}$ to zero, and hence also the righthand side of Eq.(17) (Quantum Zeno Effect). Second, the b -term in V_{eff} also rises as T^5 , so the effective matter mixing angle is very small anyway.

As T decreases with the expansion of the universe, the oscillations begin to unfreeze. Their initial non-trivial evolution is, however, still dominated by non-forward scattering. During this phase, a very interesting phenomenon will occur provided that the mild conditions discussed at the beginning of this article are met: small θ_0 and $\Delta m^2 < 0$. There is a *critical temperature* T_c , roughly given by

$$T_c \simeq (15 \rightarrow 18 \text{ MeV}) \left[\cos 2\theta_0 \frac{|\Delta m^2|}{\text{eV}^2} \right]^{\frac{1}{6}}. \quad (18)$$

Above T_c , the evolution equations drive L_{ν_α} such as to impel the effective asymmetry $L^{(\alpha)}$ towards zero (in other words, L_{ν_α} evolves from zero such as to cancel the η term in the effective asymmetry). We can call this the *asymmetry destruction phase*, and $L^{(\alpha)} = 0$ is a stable fixed point of the evolution equations. At T_c , this stable fixed point becomes unstable and runaway positive feedback leads to an exponential increase in L_{ν_α} during the *explosive growth phase*. The approximately exponential growth is cut off by the non-linear terms in the QKEs after L_{ν_α} reaches

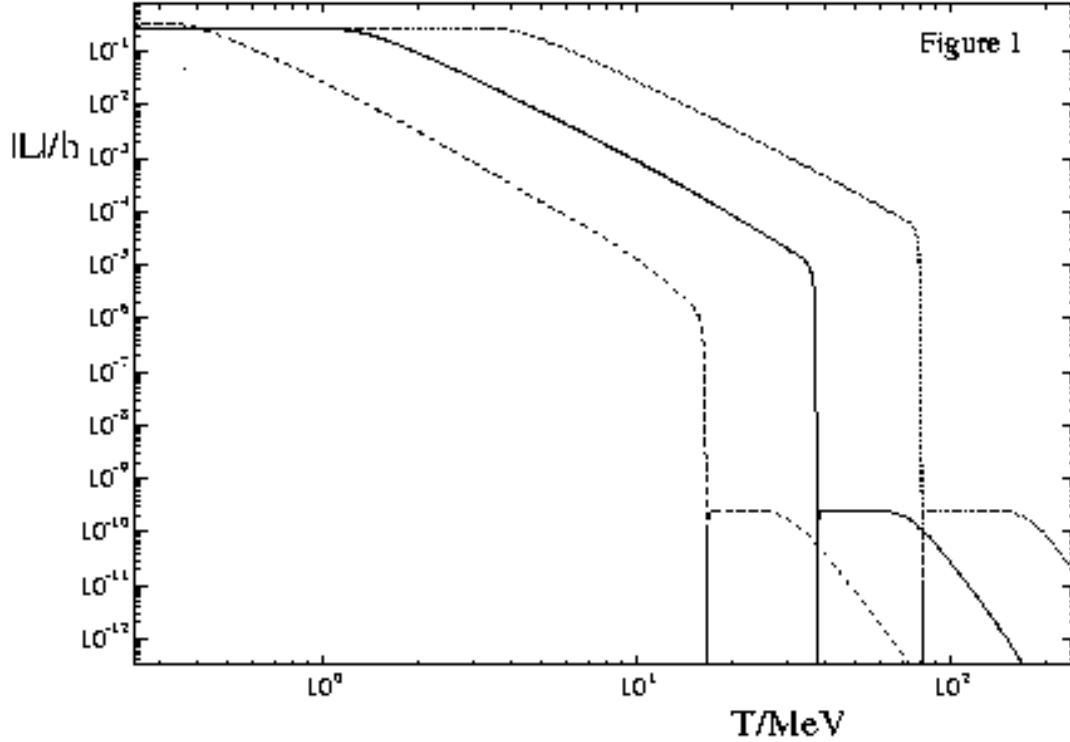


FIGURE 1. Examples of lepton asymmetry growth curves driven by $\nu_\tau \leftrightarrow \nu_s$ and the corresponding antineutrino oscillations. The mixing angle is selected to be $\sin^2 2\theta = 10^{-8}$. The three curves correspond to $\Delta m^2 = -0.5, -50, -5000 \text{ eV}^2$, reading left to right. This figure is taken from Ref. [5].

a value which is several orders of magnitude higher than the baryon and electron asymmetries. A *power law growth phase*, as per $L_{\nu_\alpha} \sim T^{-4}$, then sets in. Typically, collision dominated evolution gives way to oscillation dominated evolution during the power law phase. This is important, because physically there is a “hand over” from collisional asymmetry growth to non-linear MSW driven growth during this phase. Finally, asymmetry growth stops when L_{ν_α} reaches about $0.2 - 0.3$.

Some examples of asymmetry evolution are given in Fig.1, which is taken from Ref. [5]. These curves were produced by numerically solving the QKEs together with Eq.(17). An analytical understanding of why these curves take the displayed form has been given in the literature [1–3,13]. The key idea is to take the adiabatic limit of the QKEs. At high T , it is then possible to define precisely what is meant by collision dominated evolution, and to derive an approximate dL_{ν_α}/dt equation which makes the critical behaviour at T_c manifest. At low T , when collisions can be neglected, the adiabatic limit of the QKEs is exactly the same as the adiabatic limit for non-linear MSW evolution. See Refs. [2,3,13] for a complete discussion.

I will now review two applications, beginning with the suppression of sterile

neutrino production. Suppose that we wish to solve the atmospheric neutrino problem by maximal $\nu_\mu \rightarrow \nu_s$ oscillation with $|\Delta m_{\mu s}^2|$ in the range 10^{-3} eV² to 10^{-2} eV².³ If the $\nu_\mu + \nu_s$ subsystem does not mix with any other neutrino, then the sterile neutrinos (and antineutrinos) will certainly be brought into thermal equilibrium with the rest of the plasma prior to BBN (unless there are large pre-existing lepton asymmetries [16]). The resulting increase in the expansion rate of the universe will lead to a higher than standard Helium-4 abundance. I will assume for the sake of the example that primordial abundance observations cannot tolerate such an increase in the expansion rate. (A complete account of the somewhat complicated status of primordial abundance observations vis-à-vis BBN is beyond the scope of this talk.)

Small mixing between the ν_s and, say, a more massive ν_τ can completely change the conclusion that ν_s is brought into thermal equilibrium. The ν_τ/ν_s oscillation parameters, as chosen in the previous sentence, satisfy the requirements for large L_{ν_τ} generation. If the large L_{ν_τ} does not have its effects cancelled (see below), and if it is generated early enough, then the large matter potential consequently generated for the $\nu_\mu \leftrightarrow \nu_s$ mode will suppress these oscillations and hence also ν_s production. A numerical calculation is required to properly analyse the outcome, because the maximally mixed $\nu_\mu \leftrightarrow \nu_s$ mode always tends to induce $L^{(\mu)} \rightarrow 0$. In other words, the large L_{ν_τ} that is being created by the $\nu_\tau \leftrightarrow \nu_s$ mode could be compensated by L_{ν_μ} creation driven by $\nu_\mu \leftrightarrow \nu_s$, with the overall effect being that the μ -like effective asymmetry $L^{(\mu)}$ is driven to zero, and consequently that ν_s production through the ν_μ channel is unsuppressed after all.

Figure 2 shows the outcome of such a numerical calculation [2,5,6]. It is a plot in ν_τ/ν_s oscillation parameter space. The solid lines correspond to $|\Delta m_{\mu s}^2| = 10^{-3}, 10^{-2.5}, 10^{-2}$ eV² in ascending order up the page. Choose your favourite atmospheric $\Delta m_{\mu s}^2$. In the parameter region above the relevant solid line, the L_{ν_τ} asymmetry is *not* cancelled such that $L^{(\mu)} \rightarrow 0$. Below the solid line, it is cancelled. The transition is very sharp. So, above the solid line, ν_s production via the $\nu_\mu \rightarrow \nu_s$ mode is very suppressed. The dot-dashed line refers to ν_s production via the other available mode, $\nu_\tau \rightarrow \nu_s$. This small angle mode receives very little “self-suppression” from its own L_{ν_τ} creation activity, so its oscillation parameters must satisfy a different sort of bound. This bound is actually very similar to the old constraints calculated in Ref. [17] with asymmetry generation artificially switched off. For illustrative purposes, the dot-dashed line assumes that no more than 0.6 extra effective neutrino flavours are allowed.

The second application concerns the direct effect of a ν_e asymmetry on the neutron to proton ratio at weak freeze out. The implications of neutrino asymmetry generation in multi-flavour scenarios must be studied on a case-by-case basis. Here I will consider the scenario of Ref. [18]: there is one sterile flavour, which is used to solve the solar neutrino problem via an MSW style solution. The more massive ν_μ

³) SuperKamiokande claim that present data disfavour the $\nu_\mu \rightarrow \nu_s$ possibility relative to the $\nu_\mu \rightarrow \nu_\tau$ mode [15]. However, the dust has yet to settle on this issue.

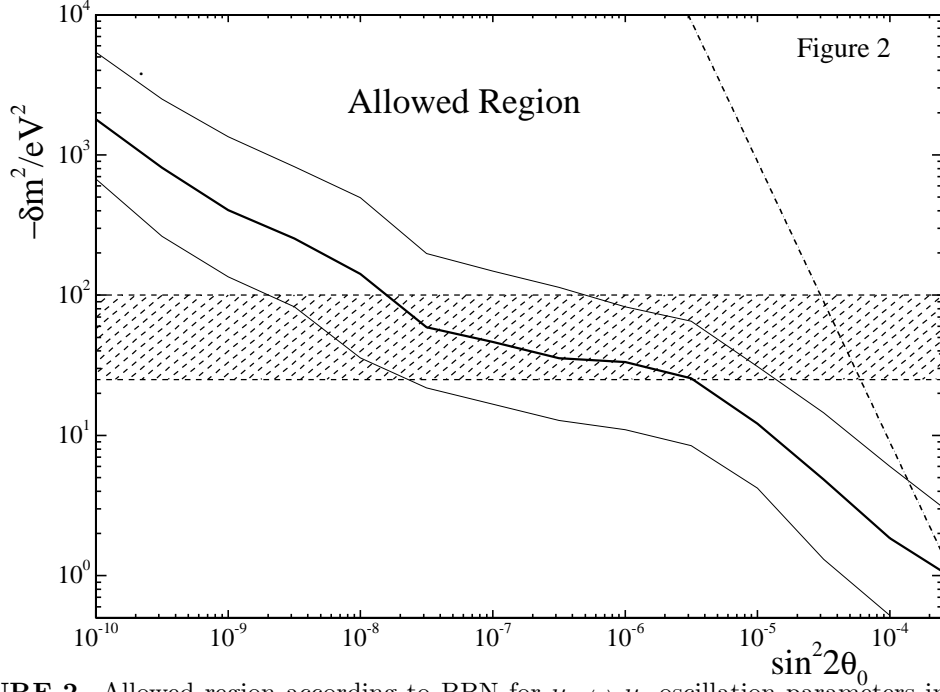


FIGURE 2. Allowed region according to BBN for $\nu_\tau \leftrightarrow \nu_s$ oscillation parameters in order to suppress maximal $\nu_\mu \leftrightarrow \nu_s$ oscillations via a large L_{ν_τ} . See text for a complete discussion. This figure is taken from Ref. [5].

and ν_τ states are maximally mixed, with the mass gap between (ν_e, ν_s) and (ν_μ, ν_τ) set in the LSND range. In this scenario, both the $\nu_\mu \rightarrow \nu_s$ and $\nu_\tau \rightarrow \nu_s$ modes satisfy the conditions for asymmetry generation. Once L_{ν_μ} and L_{ν_τ} have been generated, these asymmetries can be reprocessed into L_{ν_e} via $\nu_{\mu,\tau} \leftrightarrow \nu_e$ oscillations [8]. The L_{ν_e} outcome is insensitive to the $\nu_e/\nu_{\mu,\tau}$ mixing angles for a large range of these parameters because the MSW transitions are adiabatic. It is, however, somewhat sensitive to Δm^2 between (ν_e, ν_s) and (ν_μ, ν_τ) . Figure 3 shows a plot of the change in the effective number of neutrino flavours during BBN as a function of this Δm^2 parameter [8]. The “effective number of neutrino flavours” is just a convenient measure of the change in the Helium mass fraction Y_p relative to standard BBN, roughly obeying the relation $\delta Y_p \simeq 0.012 \delta N_\nu^{eff}$. It is affected by both expansion rate alterations and L_{ν_e} . Figure 3 incorporates both effects.

There are a few other interesting neutrino models that have been analysed in the literature. A particularly interesting case is the Exact Parity or Mirror Matter Model [11]. The full analysis is very complicated; please see Ref. [19] for all of the details.

To conclude: If light sterile or mirror neutrinos exist, then they should play a major role in cosmology. The oscillation generated neutrino asymmetry phenomenon would be a central feature. Sufficiently large neutrino asymmetries would suppress sterile or mirror neutrino production, and an electron-neutrino asymmetry of the right magnitude and sign would affect primordial Helium abundance. In particular,

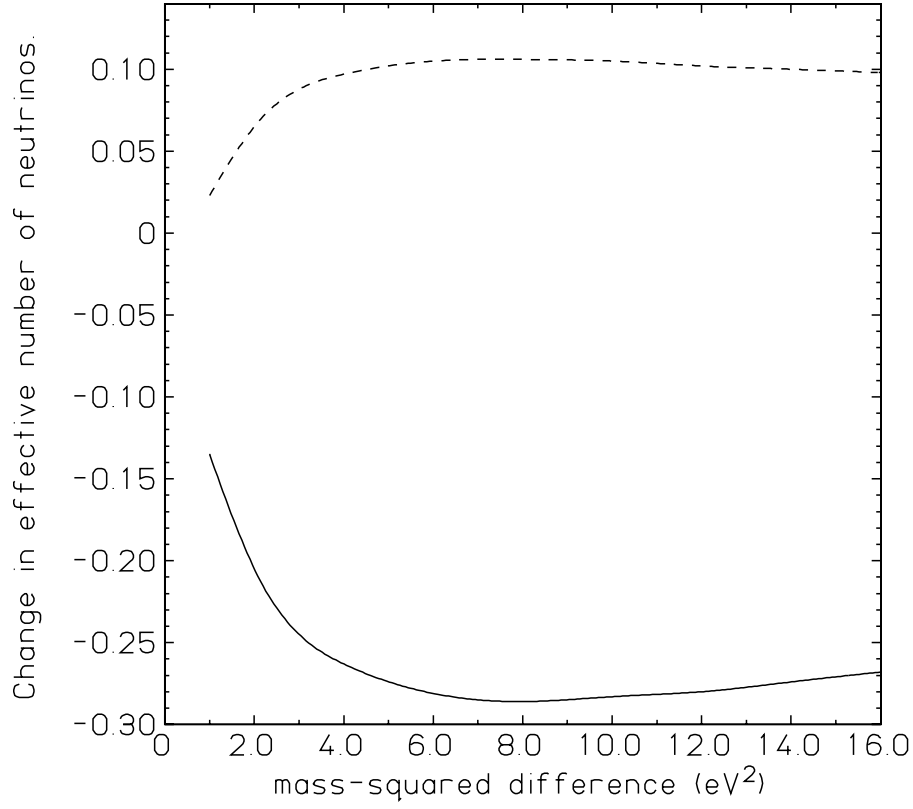


FIGURE 3. The change in the effective number of neutrino flavours during BBN for a certain neutrino scenario, plotted as a function of a Δm^2 parameter that was taken to be in the LSND range. See the text for a more complete discussion. This figure is taken from Ref. [8]

a positive L_{ν_e} would allow a larger baryon density to exist without the overproduction of Helium. We note with interest that the recent Boomerang/Maxima cosmic microwave background anistropy measurements favour a larger than standard baryon density [20]. While the combined solar, atmospheric and LSND data require at least one light sterile neutrino if oscillations are to simultaneously resolve all of the anomalies, we especially look forward to future experiments that could provide further evidence for light sterile neutrinos: SNO, MiniBooNe, the longbaseline experiments, as well as further SuperKamiokande data.

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